Lesson Twenty Four

Percentages

Aims

The aims of this lesson are to help you to:

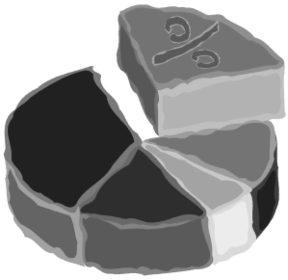
- express fractions and decimals as percentages
- explore key percentage, decimal and fraction equivalents
- work out percentage increase and decrease
- convert fractions to decimals
- calculate simple interest
- use reverse percentages

Why am I studying this?

In this lesson we're going to look at converting decimals into percentages and working out percentage increases and decreases. These skills have many practical applications in everyday life and the world of money.

Oxford Home Schooling

Fractions and Percentages



As you have seen in earlier lessons, a number that isn't whole can be expressed as a fraction of a whole number or as a decimal number. Another common way of doing this is to write the number as a percentage, which means writing it as a fraction of 100.

$\frac{100}{100}$ = 100 percent (written as 100%)

100% of something means the entire amount, so it is equal to the whole number 1.

Writing a fraction with 100 as its denominator as a percentage is fairly straightforward:

$\frac{7}{100}$ is 7% $\frac{70}{100}$ is 70%

Any percentage can easily be written as a fraction:

41% = $\frac{41}{100}$ 16% = $\frac{16}{100}$

Percentages, Decimals and Fractions

As you know, you can also express a fraction as a decimal, and it is possible to express a percentage as a decimal or a fraction, because all three represent parts of a whole (for fractions and decimals, this whole is 1; for percentages the whole is 100).

Expressing a percentage as a decimal

To do this, simply divide the percentage by 100.

Example 1 20% as a decimal is 0.2
58% as a decimal is 0.58
4% as a decimal is 0.04

Expressing a decimal as a percentage

In order to express a decimal as a percentage you simply perform the reverse of what you have just done – you multiply the decimal by 100.

Example 2 0.33 as a percentage is 33%
0.06 as a percentage is 6%
0.715 as a percentage is 71.5%

Expressing a Percentage as a Fraction

As we saw a little earlier, a percentage already is a fraction – but it is unique because, to be a percentage, it must have a denominator of 100.

So, 19% as a fraction is $\frac{19}{100}$

Remember that fractions are best expressed in their lowest form, so if you can cancel them down then you should.

Example 3 $14\% = \frac{14}{100}$

This is a fraction, but as both the numerator and the denominator are even, you can cancel it down by dividing by two.

$14\% = \frac{7}{50}$

Example 4 $20\% = \frac{20}{100}$ (This can be divided by two).

$= \frac{10}{50}$ (This can be divided by ten).

$= \frac{1}{5}$

Normally, a fraction over 100 will only divide down tidily by 2, 4, 5 or 10, or by multiples of those numbers. Cancelling by 3, or by anything else that divides by 3 (such as 6, 9, 12 or 15) will very probably produce a recurring decimal somewhere. 1/3 as a percentage is 33.3 recurring, and 1/6 is 16.6 recurring, for instance.

Expressing a Fraction as a Percentage

To write a fraction as a percentage it is easiest to first change it to a decimal, by dividing the numerator by the denominator.

Then, as shown above, you can change it to a percentage by multiplying it by 100.

Example 5 To express $\frac{4}{5}$ as a percentage, you should first divide 4 (the numerator) by 5 (the denominator).

This gives us the decimal 0.8, which, if you multiply it by 100, gives us 80.

So $\frac{4}{5} = 80\%$.

Writing One Number as a Percentage of Another

Out of a class of 16 students, 4 have read *The Lord of the Rings.* If you want to show how many students have read the book as a percentage, you first need to make a fraction out of the information. The next step is to change the fraction to a decimal. The final step is to multiply by 100 to make a percentage.

Example 6

To write 4 as a percentage of 16, first write the information as a fraction.

$\frac{4}{16}$ This can be simplified by dividing by 4.

$\frac{1}{4}$ As you already know, this is 0.25.

Multiply this by 100 and you have 25. So 25% of the class has read *The Lord of the Rings*.

Increase and Decrease of Percentages

Price rises and cuts are usually expressed as percentages. You may quite often see '20% off marked price' signs on items in a shop sale. So, if you saw a hat that was £12 and you knew that there was a 20% discount on all their hat prices, you would expect to pay 20p less for every pound,

20% of £12 = $\frac{20}{100} \times 12$ First, $\frac{20}{100} = 0.2$

So, to find 20% of 12, we need to multiply 0.2 by 12. If you need to remind yourself how to multiply decimal numbers, now would be a good time to do so by going back to Lessons 14 & 15 and re-reading those.

$$\begin{array}{r} 12 \\ \underline{\times\ 0.2} \\ 2.4 \\ \underline{000} \\ \underline{2.4} \end{array}$$

You don't need to insert the decimal point in 0.2 if you find it easier to calculate without it. We have used it in this sum to clearly show how the answer was reached.

The calculation shows that 20% of 12 is 2.4 which means that 20% of £12 is £2.40 – subtract £2.40 from £12 and you will have the discounted price of the hat.

So the sale price of the hat is £9.60.

Some Important Percentages to Learn

There are some much quicker ways of finding the percentage of a number, and it is worth memorising the ones that will prove most helpful for you.

$50\% = \frac{50}{100} = \frac{1}{2}$	So to find 50% of any number, divide the number by 2.
$25\% = \frac{25}{100} = \frac{1}{4}$	So to find 25% of a number you can divide the number by 4; alternatively you could halve the number and then halve it again.
$1\% = \frac{1}{100}$	To find 1% of a number, divide the number by 100.
$10\% = \frac{10}{100}$	To find 10% of a number, divide the number by 10.
$75\% = \frac{75}{100} = \frac{3}{4}$	To find 75% of a number, you could first find 25%, then subtract it from the original number; alternatively you could find 50% and 25% (= 'half of 50%') and add the two together.

Remember that you can always use *estimation* to check any of your answers. If a question asks you to round your answer you should leave this until the end, after you've done all the necessary calculations. This will help to keep your calculation more accurate.

Activity 1

1. Convert the following fractions into percentages:

(a) ¾

(b) 7/10

(c) 3/20

(d) 4/5

(e) 17/25 (*e.g.* as a reasonably good test score)

2. Convert the following percentages into fractions:

(a) 60%

(b) 45%

(c) 22%

(d) 94%

(e) 38%

Percentage Increase and Decrease

As we saw in the hat example, if you want to find the new amount after a percentage increase or decrease you must add to or subtract from the original figure.

Here is a simple percentage-increase question with full step-by-step working:

Example 7 A note arrives with Tom's power bill saying that the price of electricity will rise by 7% next month. If his bill is £63 at the moment, how much will Tom need to pay next month?

The first thing to do is to work out 7% of 63.

$7\% \text{ of } 63 = \frac{7}{100} \times 63$

This, as you know, works out as 0.07×63.

$$
\begin{array}{r}
0.07 \\
\times \underline{\quad 63} \\
0.21 \\
\underline{4.20} \\
\underline{4.41}
\end{array}
$$

You can check this answer using rough estimation. What is 10% of 63? (Answer: 6.3) Does the answer to the multiplication sum above still make sense?

Now you can add 4.41 to 63 to work out the new price.

63 + 4.41 = 67.41. So next month Tom would expect to pay £67.41 for his bill.

Further Percentage Increase and Decrease

A different way of working out percentage increase or decrease is to add or subtract the percentage difference *before* performing the calculation.

Example 8 A hi-fi system is on sale with 15% off its original price, which is £380. How much will the hi-fi cost?

A 15% reduction from the original price means that the hi-fi will cost 85% of £380, because 100% – 15% = 85%.

So, to work out the price of the hi-fi, we need to calculate $\frac{85}{100} \times 380$

$\frac{85}{100} = 0.85$

Remember that when you multiply decimal numbers it is often easier not to focus on the position of the decimal point until you are nearing the end of your calculation.

$$\begin{array}{r} 380 \\ \underline{0.85} \\ 1900 \\ \underline{30400} \\ \underline{32300} \end{array}$$

To work out where the decimal point should come in the answer, count the total number of decimal places in both of the numbers you multiplied.

380 is not a decimal number but 0.85 has two decimal places. Therefore, the answer you are left with is 323.00 which is the same as 323, a whole number.

This tells us that 85% of 380 is 323 and therefore £323 is the new price of the hi-fi.

Activity 2

1. I want to buy an E-pod before Christmas. Its present price is £160 … but I have heard that if I wait for the new-year sales, there will be a 15% discount. How much money will I save by waiting?

2. Meanwhile the price of fuel is going up, so it will cost more to keep our home warm during the winter. At the moment it costs £11 per week to run the heating for 7 hours a day. From January onwards it's going to cost 4% more. Work out:

   (a) By how much our monthly heating bill will rise, assuming a month to be 30 days long

   (b) What the new total rate will be (*i.e.* present rate + increase);

   (c) Alternatively, if we cut back the running time by 4% from its present 7hrs each day, how long could we then run it for the same cost as we do now?

   (d) For how much time would the heating then be *off* in a week, when previously it would still have been running?

   (e) How long would the heating be off during a typical 30-day month, compared with during previous winters?

   (f) How long would it now be off over an entire winter (6 months), again compared with before?

   (g) If we decide, after all, to keep the heating running at the same times and levels as we do now, how much more money will a whole 6-month winter cost us than before?

Converting fractions to decimals

A fraction such as $\frac{7}{100}$ is easily converted into a decimal. However we may be faced with a fraction such as $\frac{5}{8}$.

There are a couple of ways of dealing with this.

For this particular example, we could first of all find $\frac{1}{8}$ by halving $\frac{1}{4} = 0.25 = 0.250$, to give 0.125,

and then multiplying by 5 to give 0.625.

An alternative method, which can be applied to any fraction, is to write $\frac{5}{8}$ as $5 \div 8$ and carry out long division, as shown below:

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{4.800} \\ 0.200 \\ \underline{0.160} \\ 0.040 \\ \hline \end{array}$$

Simple Interest

An important practical application of percentages is in working out **interest** for money that is saved or borrowed. In everyday terms, interest is the amount that is *added* to the original sum as time passes.

One way of calculating the amount of interest earned in a savings account is as follows:

Suppose that we start with £400 in the account, and that simple interest of 5% is to be added at the end of each year. This means that after one year we add $400 \times 0.05 = £20$.

At the end of the second year, we add another 5% **of the original amount**, so that the total amount of interest earned so far is £40.

Note: As the name suggests, this is a very simple way of calculating interest, and usually a more complicated method (called compound interest) is used instead. In that case, interest is earned on the existing interest, as well as the original amount. Also, the calculation is done more frequently than each year (for example, each month; or even each day).

Example 9

£300 is paid into a savings account, and simple interest of 4% is paid each year. How much will there be in the account after ten years?

Interest of $300 \times 0.04 = £12$ will be paid at the end of each year,

so the total amount in the account at the end of ten years will be $300 + (10 \times 12) = 300 + 120 = £420$.

Activity 3	If a savings account contains £600, and simple interest of 3% is paid each year, how long will it take for the account to be more than £700?

Reverse Percentages

Example 10

A computer is on sale for £320 and the salesman says that the original price was reduced by 20%. What was that original price?

To solve this problem we can set up an equation. Let the original price be £P. Then we are being told that

$P - 0.2P = 320$

or $0.8P = 320$

Hence $P = \frac{320}{0.8} = 400$, so that the original price was £400.

Example 11

Mr Smith's council tax bill has gone up by 2% to £816. What was it before the increase?

Let the original bill be £X.

Then $1.02X = 816$ and hence $X = \frac{816}{1.02} = 800$, and the answer is £800.

Activity 4	Two new cars are on sale. Car A has been reduced by 10% to £10800 and car B has been reduced by 5% to £11495. Which car had the higher price originally?

Activity 5	Find the original prices of the following houses:
	1. House A has gone up by 5% to £126000 2. House B has gone down by 10% to £180000 3. House C has gone up by 10% to £165000 4. House D has gone down by 20% to £240000

Suggested Answers to Activities

Activity One

1.

(a) 75% (multiplied up by 25, because $4 \times 25 = 100$)

(b) 70% (multiplied by 10, 'top & bottom')

(c) 15% (multiplied by 5)

(d) 80% (multiplied by 2)

(e) 68% (multiplied by 4, like (a) inside-out)

2.

(a) 60% = 60/100 = 6/10 = 3/5

(b) 45% = 9/20 (divided by 5)

(c) 22% = 11/50

(d) 94% = 47/50

(e) 38% = 19/50

You may have noticed none of these went up or down by a factor of 3, because 3 doesn't divide cleanly into 10 or 100. (Remember those recurring decimals when we once tried something similar?)

Likewise we haven't divided the 100 into anything else that subdivides by 3, such as 6 or 9 or 12 or 15. It can be done, of course, but that would involve you handling decimal percentages: 1/3 = 33.3 (recurring) %, and 1/6 = 16.6 (recurring) %. You could try these as long divisions yourself, just to see how they run.

Activity Two

1. The E-pod will come down by 15%.

   10% of £160 is fairly obviously £16

   Another 5% will be half that much again, *i.e.* £8

   The total discount will be £16 + £8 = £24

   £160 - £24 gives a knocked-down price of £136.

2.

   (a) At £11 per week, the monthly cost will be £11 × 30/7 (to convert from weeks to months), = £330 ÷ 7 = £47.14.

   4% of this will be 4 × £47.14 ÷ 100 = £1.89 (to 2 d.p., as always for currency, and including rounding-up)

   You might alternatively have spotted that 4% = 1/25, and divided £47.14 by 25 to reach the same answer.

(b) £47.14 (old rate) + £1.89 (increase) = £49.03

(c) 7 hours = 7 × 60 mins = 420 mins

4% of 420 mins = 16.8 mins

420 mins – 16.8 mins = 403.2 mins

(= 6 hrs 43 mins 12 seconds*)

You'd be unlikely to find a heating controller that counts quite as precisely as this (and *NB*, 0.2 of a minute = 20% × 60 seconds = 2 × 6 seconds = 12 seconds). But if you were seriously concerned, cutting out a 15-minute quarter-hour somewhere from the daily program would more or less cancel out the cost of the price rise. If you were being very finicky, you might spot that this would still leave you short by 1.8 minutes a day, or 12.6 minutes a week (= 12 mins 36 sec) – so you might treat yourself to an extra 15 minutes at the weekend, or perhaps on Monday morning, to make it up even closer!

(d) A whole week's-worth of 16.8-minute daily cutbacks would amount to 117.6 minutes (117 min 36 sec, to be precise): to all practical intents this is 2 hours (120 mins). Quick rough-check: '7 lots of just over a quarter-of-an-hour = somewhere between 1¾ & 2 hours'.

(e) Keeping the accurate figure of 117.6 minutes per week, we again apply the 30/7 multiplication to scale this up for a month's-worth:

117.6 × 30/7 = 504 mins = 8.4 hrs = 8 hrs 24 mins

(Roughly, again, this is 8½ hours: more than a day's worth of difference over the entire month. You might have expected some such result, because the price rise is 4%, *i.e.* 1/25, and 25 is greater than the number of days in the month (= 30).)

(f) Six months at 8.4 hrs per month = 50.4 hours

(Rounding only a little, yet again, to convenient whole units, we find this is in the region of two whole days of constant running: 24 × 2 would have been 48hrs.)

Another way to look at it would be to say that 7 days'-worth of 7 hours each, would have been 49 hours in the week (7 × 7 = 49) which happens to be even closer still. So the price rise is virtually equivalent to having to run the system for one extra normal week over a 6-month winter.

A final rounded comparison might be to say that a whole year is roughly 50 weeks (to 1 s.f.), so half a year is roughly 25 weeks, and 4% of 25 would bring us back to one whole single week.

(g) 6 months at £1.89 more per month = £11.34

(Which is indeed much the same as an existing week's running cost; the whole winter would already be costing 6 × £47.14 = £282.84, and will now still come in at just under £300, because £282.84 + £11.34 = £294.18.

4% of £300 would only have come to £12, after all …)

Phew! But we have probably picked up some interesting number-bonds while investigating this quite realistic question – so if you've followed it right the way through, give yourself a pat on the back!

Activity Three

3% of 600 is $0.03 \times 600 = 18$

Suppose that $600 + 18n = 700$, where n is the number of years for which interest is paid. This would mean that there was exactly £700 in the account after n years.

Solving the equation, we get $18n = 100$, so that $n = \frac{100}{18} = 5.56$

Therefore, in order to have more than £700, n would need to be 6, and the answer is 6 years.

(Note that the answer has to be a whole number of years.)

Activity Four

Let original price of car A be £A.

Then $0.9A = 10800$, so that $A = \frac{10800}{0.9} = 12000$

Let original price of car B be £B.

Then $0.95B = 11495$, so that $B = \frac{11495}{0.95} = 12100$

So car B had the higher price originally.

Activity Five

1. House A has gone up by 5% to £126000

   So $1.05A = 126000$, and hence $A = \frac{126000}{1.05} = 120000$

2. House B has gone down by 10% to £180000

   So $0.9B = 180000$, and hence $B = \frac{180000}{0.9} = 200000$

3. House C has gone up by 10% to £165000

   So $1.1C = 165000$, and hence $C = \frac{165000}{1.1} = 150000$

4. House D has gone down by 20% to £240000

   So $0.8D = 240000$, and hence $D = \frac{240000}{0.8} = 300000$